UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Use laws of logarithms and remove logarithms correctly	M1	
	Obtain $x+2=e^2x$, or equivalent	A1	
	Obtain answer $x = 0.313$	A1	[3]
	[SR: If the logarithmic work is to base 10 then only the M mark is available.]		
2	*EITHER*: State correct unsimplified first two terms of the expansion of $\sqrt{(1-2x)}$, e.g. $1+\frac{1}{2}(-2x)$	B1	
	State correct unsimplified term in x^2, e.g. $\frac{1}{2}.(\frac{1}{2}-1).(-2x)^2/2!$	B1	
	Obtain sufficient terms of the product of $(1+x)$ and the expansion up to the term in x^2 of $\sqrt{(1-2x)}$	M1	
	Obtain final answer $1-\frac{3}{2}x^2$	A1	
	[The B marks are not earned by versions with symbolic binomial coefficients such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}$.]		
	[SR: An attempt to rewrite $(1+x)\sqrt{(1-2x)}$ as $\sqrt{(1-3x^2)}$ earns M1 A1 and the subsequent expansion $1-\frac{3}{2}x^2$ gets M1 A1.]		
	OR: Differentiate expression and evaluate f(0) and f ′(0), having used the product rule	M1	
	Obtain f(0) = 1 and f ′(0) = 0 correctly	A1	
	Obtain f ″(0) = –3 correctly	A1	
	Obtain final answer $1-\frac{3}{2}x^2$, with no errors seen	A1	[4]
3	Use correct quotient or product rule	M1	
	Obtain correctly the derivative in any form, e.g. $\dfrac{e^x\cos x+e^x\sin x}{\cos^2 x}$	A1	
	Equate derivative to zero and reach $\tan x = k$	M1*	
	Solve for x	M1(dep*)	
	Obtain $x=-\frac{1}{4}\pi$ (or –0.785) only (accept x in [–0.79, –0.78] but not in degrees)	A1	[5]
	[The last three marks are independent. Fallacious log work forfeits the M1*. For the M1(dep*) the solution can lie outside the given range and be in degrees, but the mark is not available if $k = 0$. The final A1 is only given for an entirely correct answer to the whole question.]		
4	State or imply $\dfrac{dx}{d\theta}=a(2-2\cos 2\theta)$ or $\dfrac{dy}{d\theta}=2a\sin 2\theta$	B1	
	Use $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\div\dfrac{dx}{d\theta}$	M1	
	Obtain $\dfrac{dy}{dx}=\dfrac{\sin 2\theta}{(1-\cos 2\theta)}$, or equivalent	A1	
	Make use of correct sin $2A$ and cos $2A$ formulae	M1	
	Obtain the given result following sufficient working	A1	[5]
	[SR: An attempt which assumes a is the parameter and θ a constant can only earn the two M marks. One that assumes θ is the parameter and a is a function of θ can earn B1M1A0M1A0.]		
	[SR: For an attempt that gives a a value, e.g. 1, or ignores a, give B0 but allow the remaining marks.]		

5	(i)	*EITHER*:	Attempt division by $2x^2 - 3x + 3$ and state partial quotient $2x$	B1	
			Complete division and form an equation for a	M1	
			Obtain $a = 3$	A1	
		*OR*1:	By inspection or using an unknown factor $bx + c$, obtain $b = 2$	B1	
			Complete the factorisation and obtain a	M1	
			Obtain $a = 3$	A1	
		*OR*2:	Find a complex root of $2x^2 - 3x + 3 = 0$ and substitute it in p(x)	M1	
			Equate a correct expression to zero	A1	
			Obtain $a = 3$	A1	
		*OR*3:	Use $2x^2 \equiv 3x - 3$ in p(x) at least once	B1	
			Reduce the expression to the form $a + c = 0$, or equivalent	M1	
			Obtain $a = 3$	A1	[3]
	(ii)		State answer $x < -\frac{1}{2}$ only	B1	
			Carry out a complete method for showing $2x^2 - 3x + 3$ is never zero	M1	
			Complete the justification of the answer by showing that $2x^2 - 3x + 3 > 0$ for all x	A1	[3]
			[These last two marks are independent of the B mark, so B0M1A1 is possible. Alternative methods include (a) Complete the square M1 and use a correct completion to justify the answer A1; (b) Draw a recognizable graph of $y = 2x^2 + 3x - 3$ or p(x) M1 and use a correct graph to justify the answer A1; (c) Find the x-coordinate of the stationary point of $y = 2x^2 + 3x - 3$ and either find its y-coordinate or determine its nature M1, then use minimum point with correct coordinates to justify the answer A1.] [Do not accept $\le$ for $<$]		
6	(i)		State or imply at any stage answer $R = 13$	B1	
			Use trig formula to find α	M1	
			Obtain $\alpha = 67.38°$ with no errors seen	A1	[3]
			[Do not allow radians in this part. If the only trig error is a sign error in $\sin(x + \alpha)$ give M1A0.]		
	(ii)		Evaluate $\sin^{-1}\left(\frac{11}{13}\right)$ correctly to at least 1 d.p (57.79577…°)	B1√	
			Carry out an appropriate method to find a value of 2θ in $0° < 2\theta < 360°$	M1	
			Obtain an answer for θ in the given range, e.g. $\theta = 27.4°$	A1	
			Use an appropriate method to find another value of 2θ in the above range	M1	
			Obtain second angle, e.g. $\theta = 175.2°$ and no others in the given range	A1	[5]
			[Ignore answers outside the given range.] [Treat answers in radians as a misread and deduct A1 from the answers for the angles.] [SR: The use of correct trig formulae to obtain a 3-term quadratic in $\tan\theta$, $\sin 2\theta$, $\cos 2\theta$, or $\tan 2\theta$ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of θ in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]		

7 (**i**) State or imply a correct normal vector to either plane, e.g. 2**i** –**j** –3**k** , or **i** + 2**j** +2**k** — B1
Carry out correct process for evaluating the scalar product of the two normals — M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result — M1
Obtain answer 57.7° (or 1.01 radians) — A1 [4]

(**ii**) *EITHER*: Carry out a complete method for finding a point on the line — M1
Obtain such a point, e.g. (2, 0, –1) — A1
EITHER: State two correct equations for a direction vector of the line, e.g. $2a - b - 3c = 0$ and $a + 2b + 2c = 0$ — B1
Solve for one ratio, e.g. $a : b$ — M1
Obtain $a : b : c = 4 : -7 : 5$, or equivalent — A1
State a correct answer, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ — A1√

OR: Obtain a second point on the line, e.g. $(0, \frac{7}{2}, -\frac{7}{2})$ — A1
Subtract position vectors to obtain a direction vector — M1
Obtain 4**i** – 7**j** + 5**k** , or equivalent — A1
State a correct answer, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ — A1√

OR: Attempt to calculate the vector product of two normals — M1
Obtain two correct components — A1
Obtain 4**i** – 7**j** + 5**k**, or equivalent — A1
State a correct answer, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ — A1√

*OR*1: Express one variable in terms of a second — M1
Obtain a correct simplified expression, e.g. $x = \dfrac{14 - 4y}{7}$ — A1
Express the first variable in terms of a third — M1
Obtain a correct simplified expression, e.g. $x = \dfrac{14 + 4z}{5}$ — A1
Form a vector equation for the line — M1
State a correct answer, e.g. $\mathbf{r} = \frac{7}{2}\mathbf{j} - \frac{7}{2}\mathbf{k} + \lambda(\mathbf{i} - \frac{7}{4}\mathbf{j} + \frac{5}{4}\mathbf{k})$, or equivalent — A1√

*OR*2: Express one variable in terms of a second — M1
Obtain a correct simplified expression, e.g. $y = \dfrac{14 - 7x}{4}$ — A1
Express the third variable in terms of the second — M1
Obtain a correct simplified expression, e.g. $z = \dfrac{5x - 14}{4}$ — A1
Form a vector equation for the line — M1
State a correct answer, e.g. $\mathbf{r} = \frac{7}{2}\mathbf{j} - \frac{7}{2}\mathbf{k} + \lambda(\mathbf{i} - \frac{7}{4}\mathbf{j} + \frac{5}{4}\mathbf{k})$, or equivalent — A1√ [6]
[The f.t. is dependent on all M marks having been obtained.]

8	**(i)**	State or obtain $\frac{dV}{dt} = 4h^2 \frac{dh}{dt}$, or $\frac{dV}{dh} = 4h^2$, or equivalent	B1	
		State or imply $\frac{dV}{dt} = 20 - kh^2$	B1	
		Use the given values to evaluate k	M1	
		Show that $k = 0.2$, or equivalent, and obtain the given equation	A1	[4]
		[The M1 is dependent on at least one B mark having been earned.]		
	(ii)	Fully justify the given identity	B1	[1]
	(iii)	Separate variables correctly and attempt integration of both sides	M1	
		Obtain terms $-20h$ and t, or equivalent	A1	
		Obtain terms $a\ln(10 + h) + b\ln(10 - h)$, where $ab \neq 0$, or $k\ln\left(\frac{10+h}{10-h}\right)$	M1	
		Obtain correct terms, i.e. with $a = 100$ and $b = -100$, or $k = 2000/20$, or equivalent	A1	
		Evaluate a constant and obtain a correct expression for t in terms of h	A1	[5]
9	**(i)**	Integrate by parts and reach $kxe^{\frac{1}{2}x} - k\int e^{\frac{1}{2}x}\,dx$	M1	
		Obtain $2xe^{\frac{1}{2}x} - 2\int e^{\frac{1}{2}x}\,dx$	A1	
		Complete the integration, obtaining $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$, or equivalent	A1	
		Substitute limits correctly and equate result to 6, having integrated twice	M1	
		Rearrange and obtain $a = e^{-\frac{1}{2}a} + 2$	A1	[5]
	(ii)	Make recognizable sketch of a relevant exponential graph, e.g. $y = e^{-\frac{1}{2}x} + 2$	B1	
		Sketch a second relevant straight line graph, e.g. $y = x$, or curve, and indicate the root	B1	[2]
	(iii)	Consider sign of $x - e^{-\frac{1}{2}x} - 2$ at $x = 2$ and $x = 2.5$, or equivalent	M1	
		Justify the given statement with correct calculations and argument	A1	[2]
	(iv)	Use the iterative formula $x_{n+1} = 2 + e^{-\frac{1}{2}x_n}$ correctly at least once, with $2 \le x_n \le 2.5$	M1	
		Obtain final answer 2.31	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (2.305, 2.315)	A1	[3]
10	**(i)**	State that the modulus of w is 1	B1	
		State that the argument of w is $\frac{2}{3}\pi$ or 120° (accept 2.09, or 2.1)	B1	[2]
	(ii)	State that the modulus of wz is R	B1√	
		State that the argument of wz is $\theta + \frac{2}{3}\pi$	B1√	
		State that the modulus of z/w is R	B1√	
		State that the argument of z/w is $\theta - \frac{2}{3}\pi$	B1√	[4]
	(iii)	State or imply the points are equidistant from the origin	B1	
		State or imply that two pairs of points subtend $\frac{2}{3}\pi$ at the origin, or that all three pairs subtend equal angles at the origin	B1	[2]

(**iv**)	Multiply 4 + 2i by w and use $i^2 = -1$	M1	
	Obtain $-(2+\sqrt{3})+(2\sqrt{3}-1)i$, or exact equivalent	A1	
	Divide 4 + 2i by w, multiplying numerator and denominator by the conjugate of w, or equivalent	M1	
	Obtain $-(2-\sqrt{3})-(2\sqrt{3}+1)i$, or exact equivalent	A1	[4]

[Use of polar form of 4 + 2i can earn M marks and then A marks for obtaining exact $x + iy$ answers.]
[SR: If answers only seen in polar form, allow B1+B1 in (**i**), B1√ + B1√ in (**ii**), but A0 + A0 in (**iv**).]